HEATING OF "THIN" BODIES IN LIQUID MEDIA

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Heating of "thin" ($Bi \leq 0.25$) bodies in liquid media is examined, taking into account crystallization and fusion at the body-medium interface.

The medium considered is a fused salt and the heating system a salt melting bath. The ratio of the volume of the bath to that of the heated element is assumed to be such that introducing the element causes a negligible drop in bath temperature; the bath temperature is constant.

The temperature difference over the section of the element when heated (or cooled) is characterized by the parameter $Bi = \alpha S/\lambda$. Distinguishing thermally "thin" bodies (i.e., those in which the temperature drop over the section can be neglected [2]) by assigning a predetermined maximum temperature difference over the section, expressed in fractions of the initial temperature difference between the liquid and the metal, we get a definite value of the Bi number, which we can use as a conventional dividing line between "thin" and "massive" bodies. In engineering calculations relating to the heating of metal, "thin" bodies are characterized by $Bi \leq 0.25$ [4].

Let us examine the heating of a flat element in a salt bath. The element may be regarded as a plane infinite wall, since two dimensions – length and width – greatly exceed the third – thickness. From the fact that the element is "thin," it follows that the temperature distribution in the element and in the film may be considered steady at any time (and, in particular, the temperature distribution in the film is linear). Heating in a salt bath is heating in a medium at a constant temperature [1, 2], but not heating at a constant surface temperature [3]. In fact, when the element is inserted into the bath, it draws heat from the surrounding fused salt and forms a layer of crystallized salt on its surface. The temperature distribution in the film, as we have noted already, is linear, increasing from the metal to the outer film surface. The latent heat of fusion of the salt and the heat it gives out in cooling from bath temperature to that of the film are acquired only by the metal. As the film grows, and the metal temperature accordingly increases, the temperature on the film outer surface increases until it reaches t_{fs}, thereafter remaining constant until the film has all fused.

Let us examine a layer of fused salt of thickness dx at distance x from the surface of the element. When the layer is cooled from temperature t_f to t_f-dt, the amount of heat given out is

$$dQ = -F\rho_s c_s dx\, dt. \qquad (1)$$

We assume here, and in what follows, that the quantities ρ_m, ρ_s, c_m, c_s, and λ_s are constant throughout the whole temperature interval studied. The error so introduced is not large, and it would not be difficult to allow for the dependence on temperature and phase state. The temperature distribution in the film obeys the equation

$$t_x = \frac{t_o - t_i}{\delta} x + t_i .$$

The amount of heat transferred to the metal by the salt in crystallizing is

$$Q_{tot} = Q_t + Q_\gamma = -\int_{t_f}^{t_x} F\rho_s c_s dt \int_0^\delta d\delta + F\rho_s \delta\gamma_s$$

or

$$Q_{tot} = F\rho_s \delta c_s \left(t_f - \frac{t_i + t_o}{2}\right) + F\delta\rho_s\gamma_s .$$

Since all the heat released by the salt in crystallizing is transferred to the metal, we have

$$c_M m_M (t_M - t_{M_0}) = F\rho_s \delta c_s \left(t_f - \frac{t_i + t_o}{2}\right) + F\rho_s\gamma_s\delta. \qquad (2)$$

Fig. 1 gives a diagram of the thermal resistances of the system. The following equations may be derived from geometrical considerations:

$$\delta = \frac{\lambda_c}{\alpha_2}\,\frac{t_o - t_{\text{в}}}{t_f - t_o}, \tag{3}$$

$$\delta = \frac{\lambda_c}{\alpha_2}\left(\frac{t_o - t_{\text{м}}}{t_f - t_o} - \frac{\alpha_2}{\alpha_1}\right). \tag{4}$$

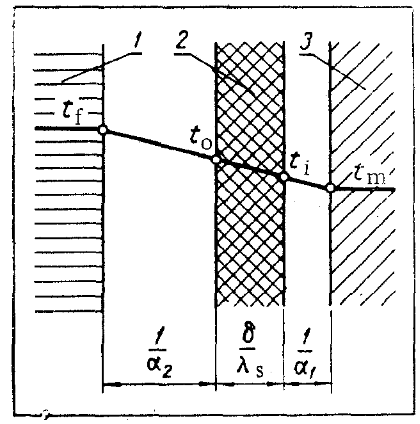

Fig. 1. Diagram of thermal resistances of system fused salt + heated body: 1 – fused salt; 2 – crystallized salt; 3 – metal element.

From (2)-(4) we can evaluate t_m and t_o as functions of δ:

$$t_{\text{м}} = \frac{2(\theta + \varphi + \delta)(t_{\text{м}_0} + \beta\delta\gamma_s) + (2\theta + \delta)t_f\beta\delta c_s}{2(\theta + \varphi + \delta) + \beta\delta c_c(2\theta + \delta)};$$

$$t_{\text{н}} = \frac{2\theta t_{\text{м}_0} + 2\theta\beta\delta\gamma_s + 2\varphi t_f + 2\delta t_f + 2\theta\beta\delta c_s t_f + \beta\delta^2 c_s t_f}{2(\theta + \delta + \varphi) + \beta\delta c_s(2\theta + \delta)};$$

$$\theta = \frac{\lambda_s}{\alpha_2};\quad \varphi = \frac{\lambda_s}{\alpha_1};\quad \beta = \frac{F\rho_s}{c_{\text{м}} m_{\text{м}}} = \frac{F\rho_s}{c_{\text{м}}\frac{F}{2}S\rho_{\text{м}}} = \frac{2\rho_s}{c_{\text{м}} S\rho_{\text{м}}}.$$

Putting $t_o = t_{fs}$, we obtain the maximum film thickness from (6)

$$\begin{aligned}\delta_0 &= (-b + \sqrt{b^2 - 4al})/2a;\\ a &= \beta c_s(t_f - t_{fs})\cdot 1/2;\\ b &= \theta\beta c_s(t_f - t_{fs}) + \theta\beta\gamma_s + t_f - t_{fs};\\ e &= \varphi(t_f - t_{fs}) - \theta(t_{fs} - t_{\text{м}_0}).\end{aligned} \tag{6'}$$

It follows from (4) that the film will be completely fused ($\delta = 0$) when the metal temperature $t_{cff} = t_{fs} - (t_f - t_o)\frac{\alpha_2}{\alpha_1}$. If $\alpha_1 \gg \alpha_2$ we have the trivial case $t_m = t_{fs}$.

Let us now find the growth time for the film. At any time τ during growth, the amount of heat passing through the outer surface of the crystallized film in time $d\tau$ is given by Newton's law:

$$dQ_g = F\alpha_2(t_f - t_o)\,d\tau.$$

This amount of heat may be represented as the amount of heat given up by a layer of crystallizing salt of thickness $d\delta$ in cooling from t_f to t_o, plus the latent heat of fusion of the layer released in crystallizing:

$$F\alpha_2(t_f - t_o)\,d\tau = F\rho_s c_s(t_f - t_o)\,d\delta + F\rho_s\gamma_s\,d\delta \tag{7}$$

or

$$d\tau = \frac{\rho_s c_s}{\alpha_2}\,d\delta + \frac{\rho_s\gamma_s}{\alpha_2}\,\frac{d\delta}{t_f - t_o}. \tag{8}$$

By first substituting the value $t_f - t_o$ from (6), and then integrating (8) in the limits 0 to τ_g and 0 to δ, we obtain the time taken for a film of thickness δ to form. For $\delta = \delta_0$, we find the total time of film growth τ_{g0}.

$$\tau_g = \left[\frac{\rho_s(\theta + \varphi)}{\lambda_s\beta} + \frac{\rho_s(1 + \theta\beta c_s)(t_f - t_{\text{м}_0})}{\lambda_s\beta^2\gamma_s} + \frac{\rho_s c_s(t_f - t_{\text{м}_0})^2}{2\lambda_s\beta^2\gamma_s^2}\right]\ln\frac{t_f - t_{\text{м}_0}}{t_f - t_{\text{м}_0} - \beta\gamma_s\delta} - \left[\frac{\rho_s c_s(t_f - t_{\text{м}_0})}{2\lambda_s\beta\gamma_s} + \frac{\rho_s}{\lambda_s\beta}\right]\delta - \frac{\rho_s c_s}{4\lambda_s}\delta^2. \tag{9}$$

To determine the fusion time of the film, we note that, after the temperature of the outer surface of the film has attained the fused salt temperature, the heat incident on the outer surface of the film in time $d\tau$ will go to increase the metal temperature and the film temperature, and to transfer latent heat of fusion to the melting layer of the film $d\delta$.

$$F\alpha_2(t_f - t_o)\,d\tau = c_{\text{м}} m_{\text{м}}\,dt_{\text{м}} + F\rho_s c_s\delta\,d\left(\frac{t_o + t_i}{2}\right) - F\rho_s\gamma_s\,d\delta. \tag{10}$$

The last term is negative, since in this case the latent heat is absorbed.

Since $t_o = t_{fs}$ during the second period of the existence of the film, we may determine dt_i and dt_m from (3) and (4), and, substituting in (10), we have the differential equation for the film fusion time:

$$d\tau_{cf} = -\frac{\rho_s}{\theta\alpha_2\beta}\,d\delta - \frac{\rho_s c_s}{\alpha_2(\theta+\varphi)}\,\delta\frac{d\delta}{2} - \frac{\gamma_s\rho_s}{\alpha_2(t_f - t_{fs})}\,d\delta. \tag{11}$$

Integrating within the limits τ_{g0} to τ_{cf} and from δ_0 to δ, we find the time required to melt the film from δ_0 to δ:

$$\Delta\tau_{cf} = \tau_{cf} - \tau_{go} = \left[\frac{\rho_s}{\theta\alpha_2\beta} + \frac{\gamma_s\rho_s}{\alpha_2(t_f - t_{fs})}\right](\delta_0 - \delta) + \frac{\rho_s c_s}{4\alpha_2(\theta+\varphi)}(\delta_0^2 - \delta^2). \tag{12}$$

Putting $\delta = 0$ we obtain the time for complete fusion of the film:

$$\tau_{cf.o} = \tau_{go} + \left[\frac{\rho_s}{\lambda_s\beta} + \frac{\gamma_s\rho_s}{\alpha_2(t_f - t_{fs})}\right]\delta_0 + \frac{\rho_s c_s}{4\alpha_2(\theta+\varphi)}\delta_0^2. \tag{13}$$

We see from (9) and (13) that the time of existence of the film, from the start of heating to the end of fusion is

$$\tau_{cf.o} = \left[\frac{\gamma_s\rho_s}{\alpha_2(t_f - t_{fs})} - \frac{\rho_s c_s(t_f - t_{M_0})}{2\lambda_s\beta\gamma_s}\right]\delta_0 - \left[\frac{\rho_s c_s}{4\lambda_s}\left(1 - \frac{\alpha_1}{\alpha_1+\alpha_2}\right)\right]\delta_0^2 + \left[\frac{\rho_s(\theta+\varphi)}{\lambda_s\beta} + \frac{\rho_s(1+\theta\beta c_s)(t_f - t_{M_0})}{\lambda_s\beta^2\gamma_s} + \frac{\rho_s c_s(t_f - t_{M_0})^2}{2\lambda_s\beta^2\gamma_s^2}\right]\ln\frac{t_f - t_{M_0}}{t_f - t_{M_0} - \beta\gamma_s\delta_0}$$

The time for the element to be heated from t_{mcf} to t_{mf} may be determined from the known heating time of "thin" bodies, [1]:

$$\Delta\tau_f = \tau_{tot} - \tau_{cf.o} = \frac{\rho_M S c_M}{2\alpha_2}\ln\frac{t_f - t_{mcf}}{t_f - t_{mf}}. \tag{14}$$

The heat transfer coefficients α_1 and α_2 in (9), (13), and (14) are different, since they refer to different temperature intervals. This difference can be taken into account, but for simplicity we shall take average values of α_1 and α_2 over the whole temperature range in question, t_{mo} to t_{mf}.

Therefore, the time required for the element to be heated in the liquid from t_{mo} to t_{mf} is

$$\tau_{tot} = \left[\frac{\gamma_s\rho_s}{\alpha_2(t_f - t_{fs})} - \frac{\rho_s c_s(t_f - t_{M_0})}{2\lambda_s\beta\gamma_s}\right]\delta_0 - \left[\frac{\rho_s c_s}{4\lambda_s}\left(1 - \frac{\alpha_1}{\alpha_1+\alpha_2}\right)\right]\delta_0^2 + \left[\frac{\rho_s(\theta+\varphi)}{\lambda_s\beta} + \frac{\rho_s(1+\theta\beta c_s)(t_f - t_{M_0})}{\lambda_s\beta^2\gamma_s} + \frac{\rho_s c_s(t_f - t_{M_0})^2}{2\lambda_s\beta^2\gamma_s^2}\right]\ln\frac{t_f - t_{M_0}}{t_f - t_{M_0} - \beta\gamma_s\delta_0} + \frac{\rho_M c_M S}{2\alpha_2}\ln\frac{t_f - t_{mcf}}{t_f - t_{mf}}. \tag{15}$$

The above calculation is correct for bodies that are "thin" from the thermal point of view. Since it also follows that the crystallized salt film must be thermally "thin" ($Bi = \alpha_2\delta_0/\lambda_s \le 0.25$), knowing δ_0 and Bi, we obtain the thickness S of the element on which a thermally "thin" film of thickness δ_0 is formed:

$$S \leqslant \frac{0{,}5\rho_s}{c_M\rho_M}\left(\frac{\lambda_s}{\alpha_2}\right)^2 \frac{1{,}5c_s(t_f - t_{fs}) + \gamma_s}{\frac{\lambda_s}{\alpha_2}(t_{fs} - t_{M_0}) - \frac{\lambda_s}{\alpha_1}(t_f - t_{fs}) - 0{,}25\frac{\lambda_s}{\alpha_2}(t_f - t_{fs})}. \tag{16}$$

For "thin" elements, moreover, we must have the relation $Bi = \alpha_2 S/\lambda_m \leq 0.25$. An element which is thermally "thin" for heating in a given liquid will have a thickness which satisfies both relationships. The above method of calculating heating times in liquids will be valid precisely for such elements.

To illustrate the results obtained, we shall calculate the heating time for an element with $S = 2.10^{-3}$ m. The medium is fused NaCl, with $t_f = 1,273°K$; $t_{mf} = 1,173°K$; $t_{m_0} = 293°K$; $t_{fs} = 1,077°K$ [5]; $\rho_s = 2,160$ kg/m^3 [5]; $\rho_m = 7,800$ kg/m^3 [2]; $\gamma_s = 5.15 \cdot 10^5$ j/kg [5]; $c_m = 0.5 \cdot 10^3$ j/kg · degree [2]; $c_s = 0.88 \cdot 10^3$ j/kg · degree [5]; $\lambda_m = 30.4$ w/m · degree [2]; $\lambda_s = 1.12$ w/m · degree [7]; $\alpha_2 = 350$ w/m^2 · degree [2].

No accurate data on α_1 are given in the literature. We deduce from [7] that α_1 must be of the order 1,163 w/m^2 · degree.

From (6'), $\delta_0 = 0.574 \cdot 10^{-3}$ m; for the element $Bi = 1.15 \cdot 10^{-2} \leq 0.25$; for the film $Bi = 0.18 < 0.25$; $\tau_{tot} = 43.9$ sec.

Practical recommendations as regards the heating time for rectangular elements in salt baths at such temperatures [8] indicate a value of τ_{tot} of 18-22 sec. for a 1 mm section, which agrees reasonably with the calculated value.

NOTATION

α_1 and α_2 — heat transfer coefficients for film/metal and fused salt/metal (or fused salt/film); λ_m and λ_s — thermal conductivity of metal and crystallized salt; t_f, t_{fs}, t_o and t_i, t_{m_0} and t_{mf} — temperatures of: medium or fused salt, fusion of salt, outer and inner film surface, metal — initial and final; s — calculated thickness of element; F — surface area of element; m — mass of element; ρ_m — density of metal; ρ_s — density of crystallized salt; c_m and c_s — specific heat capacity of metal and salt; δ — thickness of film of crystallized salt; γ_s — latent heat of fusion of salt.

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